Nonlinear Theory of Soliton-Induced Waveguides

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We develop a nonlinear theory of soliton-induced waveguides that describes a finite-amplitude probe beam guided by a spatial dark soliton, in a saturable nonlinear medium. We suggest an effective way of controlling the interaction of these soliton-induced waveguides and also show that, in sharp contrast with scalar dark solitons, the dark-soliton waveguides can attract each other and even form stationary bound states.

It was suggested in the early days of nonlinear optics, that the waveguiding properties of a self-trapped laser beam could be used to guide atoms or ions along the beam's axis [1]. A similar physical mechanism is responsible for the trapping of a weak beam by a spatial (bright or dark) optical soliton. This phenomenon is usually referred to as the soliton-induced waveguiding of a probe beam. It has been widely discussed theoretically and has also been observed experimentally [2–6].

As has already been established, the major advantage of employing the waveguiding properties of dark, rather than bright, spatial solitons is their greater stability and steerability in a Kerr medium [7]. Moreover, in the case of saturable nonlinearity, dark-soliton waveguides remain single-moded [8], unlike the waveguides created by bright solitons [6].

The theory of soliton-induced waveguides developed so far is based on the assumption that a probe beam guided by a spatial soliton is *weak* (see, e.g., Refs. [3,4,6]). However, when the intensity of the probe beam is increased, the guided mode requires a broader waveguide and the beam interacts strongly with the waveguide, changing it in a self-consistent manner. Such a structure can not be described by a linear waveguide theory because it represents a bound state of the coupled bright and dark components. In this Letter we develop a nonlinear theory of the soliton-induced waveguides, treating the case of two incoherently interacting beams in a photorefractive medium as a model already realized experimentally [9]. We also analyze the interaction of two neighboring waveguides and show that attractive forces acting between the guided beams can result in suppression of the repulsion between dark solitons.

Following the original paper by Christodoulides *et al.* [10], we consider two incoherently interacting linearly polarized beams with the normalized envelopes U and W in a photorefractive medium, described by the equations,

$$\begin{aligned} i\frac{\partial U}{\partial z} + \frac{1}{2}\frac{\partial^2 U}{\partial x^2} + \frac{\beta(1+\rho)U}{1+|U|^2+|W|^2} &= 0, \\ i\frac{\partial W}{\partial z} + \frac{1}{2}\frac{\partial^2 W}{\partial x^2} + \frac{\beta(1+\rho)W}{1+|U|^2+|W|^2} &= 0. \end{aligned} \tag{1}$$

Here $\rho = I_\infty/I_d$, $\beta = (k_0 x_0)^2 n_e^4 r_{33} E_0/2$, where I_∞ stands for the total power density away from the beam, I_d is the so-called dark irradiance, k_0 is the propagation constant, x_0 is the spatial width of the beam, and $n_e^4 r_{33} E_0$ is a correction to the refractive index that it due to the external field applied to a crystal along the x-axis [10].

Localized solutions of Eqs. (1) can be sought in the form $U = \sqrt{1-q_1}\, u\, e^{iq_1 z}$ and $W = \sqrt{1-q_1}\, w\, e^{iq_2 z}$, and they form a continuous two-parameter family. System (1) can be further simplified by measuring the spatial coordinates z and x in units of $\sqrt{s}/\beta(1+\rho)$ and $\{s/[\beta(1+\rho)]\}^{1/2}$, respectively, where $s = 1 - q_1$. Then, u and w satisfy the normalized equations:

$$\begin{aligned} i\frac{\partial u}{\partial z} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} - \frac{(|u|^2+|w|^2)u}{1+s(|u|^2+|w|^2)} + u &= 0, \\ i\frac{\partial w}{\partial z} + \frac{1}{2}\frac{\partial^2 w}{\partial x^2} - \frac{(|u|^2+|w|^2)w}{1+s(|u|^2+|w|^2)} + \lambda w &= 0, \end{aligned} \tag{2}$$

where $\lambda = (1-q_2)/s < 1$ is the dimensionless soliton parameter, and s characterizes *nonlinearity saturation*, ($s < 1$). In the limit $s \to 0$, Eqs. (2) reduce to the Manakov model which possesses exact bright and dark multisoliton solutions [11–14].

Stationary solutions are described by Eqs. (2) with z derivatives omitted. We assume that the component u has nonvanishing asymptotics, whereas w is spatially localized. Then the simplest solution of this kind describes a dark soliton of the field u, provided that $w \equiv 0$. This solution can be presented in quadratures and then found numerically [6]. For the component w, the dark soliton creates an effective waveguide. To analyze what kind of guided mode can be supported by this waveguide we linearize the equation for w and study its localized solutions. That is, we consider a one-component dark soliton $u = u_s(x)$ and add a small perturbation written as $w = \mathrm{O}(\epsilon)$ and $u = u_s(x) + \mathrm{O}(\epsilon^2)$. It is easy to verify from Eqs. (2) that $\mathrm{O}(\epsilon)$ in the expansion for u is zero. Substituting this expansion into Eqs. (2), we obtain two decoupled equations, of which the stationary equation for w is

$$\frac{1}{2}\frac{d^2 w}{dx^2} - \frac{|u_s|^2}{1+s|u_s|^2} w + \lambda w = 0. \tag{3}$$

Depending on the properties of the effective waveguide $V(x) \equiv |u_s(x)|^2(1+s|u_s(x)|^2)^{-1}$, eigenvalue problem (3) possesses a number of solutions that decay as $x \to \pm\infty$, the so-called *guided* or *bound* modes. In the limit $s \to 0$,

the symmetric solution $w_1(x) = \operatorname{sech} x$ exists for any $\lambda > \lambda_0 = 1/2$. In general, the cutoff of a bound mode, $\lambda_0(s)$, depends on the saturation parameter s. Furthermore, if $u_s(x)$ is a dark soliton of a saturable medium, there exists *only one bound mode* of eigenvalue problem (3) for any value of $s < 1$, as was also found in Ref. [6]. Therefore the dark-soliton waveguide in a saturable medium is always single moded [8].

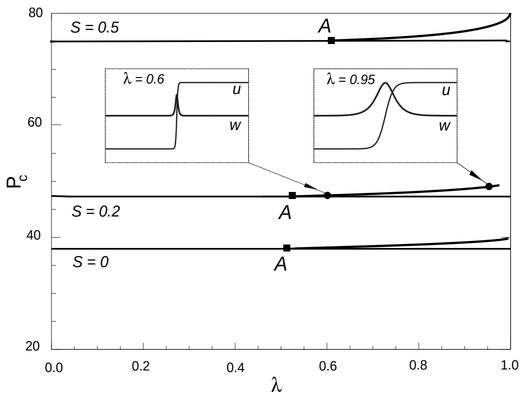

FIG. 1. Bifurcation of a one-component dark soliton $u_s(x)$ for three values of s, shown as the dependence of the complimentary power P_c on the propagation constant λ. Examples of two-component localized solutions are shown for $s = 0.2$.

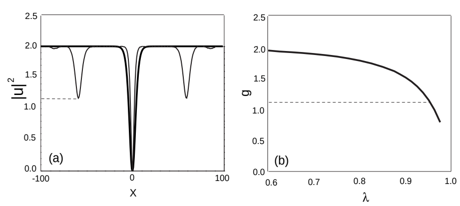

FIG. 2. Evolution of the dark-soliton waveguide without the guided mode for $s = 0.5$. (a) Soliton profiles at $z = 0$ (thick line) and $z = 100$ (thin line) for $\lambda = 0.95$. (b) Amplitude of the additional gray-soliton pair g versus λ measured at $z = 100$.

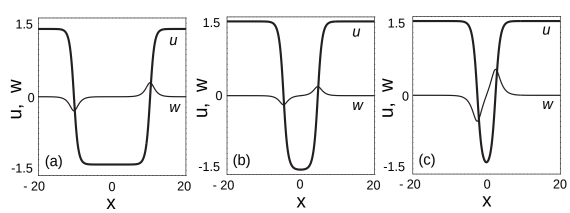

FIG. 3. Examples of two-soliton bound states in model (2): (a) $s = 0.4$ and $\lambda = 0.55$, (b) $s = 0.5$ and $\lambda = 0.55$, (c) $s = 0.5$ and $\lambda = 0.65$.

The localized solution that describes the fundamental (symmetric) mode of a dark-soliton waveguide appears as a bifurcation of the one-component dark soliton, u_s, in which a two-component solution (u, w) emerges from the one-component solution $(u_s, 0)$ at certain $\lambda = \lambda_0(s)$. Such localized solutions form a continuous family in λ for a fixed s. Figure 1 presents examples of such a bifurcation together with the corresponding two-component soliton states, found numerically by the relaxation technique, for different values of s. Shown is the dependence of the complimentary power,

$$P_c = \int_{-\infty}^{+\infty} dx \left\{ |u(x)|^2 + |w(x)|^2 - (1-s)^{-1} \right\},$$

on the soliton parameter, λ.

From the physical point of view, the parameter λ characterizes the intensity of a bright component guided by a dark-soliton waveguide. As λ grows, the intensity of the bright component grows as well, remaining less than that of the dark-soliton background. As it approaches a maximum, the dark soliton becomes broader. This means that, for λ much larger than $\lambda_0(s)$, the soliton waveguide deforms: It becomes wider to guide a beam of higher intensity. This effect is definitely beyond the linear approximation provided by the model [Eq. (3)]. It can, however, be described by a nonlinear theory constructed by numerical solution of coupled equations (2). The corresponding solutions, presented in Fig. 1 for $\lambda > \lambda_0(s)$, generalize the localized solutions of the linear eigenvalue problem [Eq. (3)] valid only near the bifurcation points A where $\lambda \sim \lambda_0$.

To verify the concept of a deformed waveguide, we performed simulations of the waveguide propagation in the case when the bright component is removed. As expected from the theory of dark solitons [7] and previously confirmed in [15], a wider dip in the input beam generates at least one additional pair of gray solitons, as is shown in Fig. 2(a). The amplitude of the additional gray-soliton pair g is shown in Fig. 2(b) as function of the parameter λ.

Similar to the case of the exactly integrable Manakov system [13,14], the nonintegrable model (2) supports stationary bound states of two (or more) solitons. We have found such stationary structures numerically. Examples of the stationary solutions, corresponding to different values of λ and s, are shown in Figs. 3 (a,b,c).

The stationary bound states are expected when the total force acting between the neighboring two-component solitons vanishes. Two closely separated dark solitons always repel each other, and this property does not depend on integrability [7]. In the light of this knowledge, the existence of stationary solutions in the form of two bound dark-soliton waveguides is rather surprising. Therefore, we are led to wonder whether the introduction of a bright component causes *an attractive force* between solitons that could nullify the repulsion of the dark components. Note that the stationary bound states of the model [Eqs. (2)] are formed by bright components that are *out of phase* with each other (Figs. 3). Expecting them to attract each other would be inconsistent with the theory of bright solitons that predicts attraction only between in-phase solitons. However, in our case the bright component by itself is not a soliton because, after the dark component is removed, it diffracts in the defocusing medium. Localization of the bright components is due to trapping

in an effective waveguide created by dark components. As a result, interaction of these guided modes is different to that expected for bright solitons. From the physical point of view, even slight overlap between two out of phase localized beams in a defocusing medium leads to an increase of the refractive index in the overlapping region, therefore increasing the attraction of the beams.

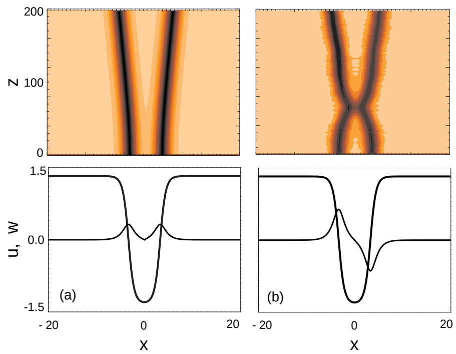

FIG. 4. Interaction between two dark-soliton waveguides guiding bright components (a) in phase or (b) out of phase.

Analyzing interaction between the soliton waveguides, we look for a solution to describe two weakly overlapping two-component solitons in the form: $u = \tanh(x - x_0)\tanh(x + x_0)$ and $w = a\,\mathrm{sech}(x - x_0) + a\,e^{i\phi}\mathrm{sech}(x + x_0)$. Then we employ the Lagrangian formalism for dark solitons [16] to derive effective equations for the soliton parameters, including the soliton separation $2x_0$ and the relative phase between the bright components ϕ. Omitting lengthy details, we present the effective energy of the soliton interaction in the form ($x_0 \gg 1$),

$$U_{\rm eff}(x_0) = 2e^{-4x_0} + 4a^2\, x_0 e^{-2x_0}\cos(\phi). \qquad (4)$$

The first term in Eq. (4) describes the interaction of two scalar dark solitons; see Eq. (34) in Ref. [16]. The second term appears as the result of the interaction between the bright components of the amplitude a. As follows from Eq. (4), the repulsive force between dark components is *weaker* than the force acting between the bright components. Moreover, introducing bright components into the closely separated dark solitons can lead to a nontrivial effect when the repulsion of the dark components is compensated for by an attractive force acting between the bright components. This effect requires that the bright components be out of phase ($\phi = \pi$), in agreement with the stationary solutions found numerically; see Fig. 3. Figure 4 shows the results of the propagation of two neighboring dark-soliton waveguides when the bound modes they guide are in phase [Fig. 4(a)] or out of phase [Fig 4(b)]. In the former case the interaction between bright components is repulsive, and the solitons repel each other even more strongly than two scalar dark solitons. In the latter case, the interaction is attractive and it forces the dark solitons to collide. These results confirm that the interaction of the dark-soliton waveguides is phase sensitive. We believe that the demonstrated behaviour can be observed experimentally for the incoherent interaction of photorefractive solitons.

In conclusion, we have developed a nonlinear theory of the soliton-induced waveguides created by dark solitons in a saturable optical medium. We have found families of the soliton states describing a finite-amplitude guided mode trapped by a dark soliton. We have described a novel type of interaction of the dark-soliton waveguides in which the repulsion of the neighboring dark solitons is suppressed by an attractive force acting between the guided modes, leading to the existence of stationary two-soliton bound states.

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